LAWS OF EXPONENTS 3.1.1 and 3.1.2

In general, to simplify an expression that contains exponents means to eliminate parentheses and negative exponents if possible. The basic **laws of exponents** are listed here.

(1) $x^a \cdot x^b = x^{a+b}$

Examples: $x^3 \cdot x^4 = x^7$; $2^7 \cdot 2^4 = 2^{11}$

(2) $\dfrac{x^a}{x^b} = x^{a-b}$

Examples: $\frac{x^{10}}{x^4} = x^6$; $\frac{2^4}{2^7} = 2^{-3}$

(3) $\left(x^a\right)^b = x^{ab}$

Examples: $\left(x^4\right)^3 = x^{12}$; $\left(2x^3\right)^5 = 2^5 \cdot x^{15} = 32x^{15}$

(4) $x^0 = 1$

Examples: $2^0 = 1$; $(-3)^0 = 1$; $\left(\frac{1}{4}\right)^0 = 1$

(5) $x^{-n} = \dfrac{1}{x^n}$

Examples: $x^{-3} = \frac{1}{x^3}$; $y^{-4} = \frac{1}{y^4}$; $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

(6) $\dfrac{1}{x^{-n}} = x^n$

Examples: $\frac{1}{x^{-5}} = x^5$; $\frac{1}{x^{-2}} = x^2$; $\frac{1}{3^{-2}} = 3^2 = 9$

(7) $x^{m/n} = \sqrt[n]{x^m}$

Examples: $x^{2/3} = \sqrt[3]{x^2}$; $y^{1/2} = \sqrt{y}$

In all expressions with fractions we assume the denominator does not equal zero.

For additional information, see the Math Notes box in Lesson 3.1.2. For additional examples and practice, see the Checkpoint 5A problems in the back of the textbook.

Example 1

Simplify: $\left(2xy^3\right)\left(5x^2y^4\right)$

Reorder: $2 \cdot 5 \cdot x \cdot x^2 \cdot y^3 \cdot y^4$

Using law (1): $10x^3y^7$

Example 2

Simplify: $\dfrac{14x^2y^{12}}{7x^5y^7}$

Separate: $\left(\dfrac{14}{7}\right) \cdot \left(\dfrac{x^2}{x^5}\right) \cdot \left(\dfrac{y^{12}}{y^7}\right)$

Using laws (2) and (5): $2x^{-3}y^5 = \dfrac{2y^5}{x^3}$

Example 3

Simplify: $\left(3x^2y^4\right)^3$

Using law (3): $3^3 \cdot \left(x^2\right)^3 \cdot \left(y^4\right)^3$

Using law (3) again: $27x^6y^{12}$

Example 4

Simplify: $\left(2x^3\right)^{-2}$

Using law (5): $\dfrac{1}{\left(2x^3\right)^2}$

Using law (3): $\dfrac{1}{2^2 \cdot \left(x^3\right)^2}$

Using law (3) again: $\dfrac{1}{4x^6}$

Example 5

Simplify: $\dfrac{10x^7y^3}{15x^{-2}y^3}$

Separate: $\left(\dfrac{10}{15}\right)\cdot\left(\dfrac{x^7}{x^{-2}}\right)\left(\dfrac{y^3}{y^3}\right)$

Using law (2): $\dfrac{2}{3}x^9y^0$

Using law (4): $\dfrac{2}{3}x^9 \cdot 1 = \dfrac{2}{3}x^9 = \dfrac{2x^9}{3}$

Problems

Simplify each expression. Final answers should contain no parentheses or negative exponents.

1. $y^5 \cdot y^7$

2. $b^4 \cdot b^3 \cdot b^2$

3. $8^6 \cdot 8^{-2}$

4. $(y^5)^2$

5. $(3a)^4$

6. $\dfrac{m^8}{m^3}$

7. $\dfrac{12m^8}{6m^{-3}}$

8. $(x^3y^2)^3$

9. $\dfrac{(y^4)^2}{(y^3)^2}$

10. $\dfrac{15x^2y^5}{3x^4y^5}$

11. $(4c^4)(ac^3)(3a^5c)$

12. $(7x^3y^5)^2$

13. $(4xy^2)(2y)^3$

14. $\left(\dfrac{4}{x^2}\right)^3$

15. $\dfrac{(2a^7)(3a^2)}{6a^3}$

16. $\left(\dfrac{5m^3n}{m^5}\right)^3$

17. $(3a^2x^3)^2(2ax^4)^3$

18. $\left(\dfrac{x^3y}{y^4}\right)^4$

19. $\left(\dfrac{6x^8y^2}{12x^3y^7}\right)^2$

20. $\dfrac{(2x^5y^3)^3(4xy^4)^2}{8x^7y^{12}}$

21. x^{-3}

22. $2x^{-3}$

23. $(2x)^{-3}$

24. $(2x^3)^0$

25. $5^{1/2}$

26. $\left(\dfrac{2x}{3}\right)^{-2}$

Core Connections Algebra

Answers

1. y^{12}
2. b^9
3. 8^4
4. y^{10}
5. $81a^4$
6. m^5
7. $2m^{11}$
8. x^9y^6
9. y^2
10. $\frac{5}{x^2}$
11. $12a^6c^8$
12. $49x^6y^{10}$
13. $32xy^5$
14. $\frac{64}{x^6}$
15. a^6
16. $\frac{125n^3}{m^6}$
17. $72a^7x^{18}$
18. $\frac{x^{12}}{y^{12}}$
19. $\frac{x^{10}}{4y^{10}}$
20. $16x^{10}y^5$
21. $\frac{1}{x^3}$
22. $\frac{2}{x^3}$
23. $\frac{1}{8x^3}$
24. 1
25. $\sqrt{5}$
26. $\frac{9}{4x^2}$

EQUATIONS ↔ ALGEBRA TILES 3.2.1

An Equation Mat can be used together with algebra tiles to represent the process of solving an equation. For assistance with Lesson 3.2.1, see Lessons A.1.1 through A.1.9 in this *Parent Guide with Extra Practice*.

See the Math Notes box in Lesson A.1.8 (in the Appendix chapter of the textbook) and in Lesson 3.2.1 for a list of all the "legal" moves and their corresponding algebraic equivalents. Also see the Math Notes box in Lesson A.1.9 (in the Appendix chapter of the textbook) for checking a solution.

For additional examples and practice, see the Checkpoint 1 materials at the back of the textbook.