Everyday Math Skills

Simply Math

Everyday Math Skills | 2009

Northwest Territories Education, Culture and Employment

NWTLC
NWT Literacy Council

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Northwest Territories Education, Culture and Employment

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Contact the NWT Literacy Council to get copies of the *Simply Math Booklet*. Or you can download it from our website.

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Table of Contents

How to Use This Book

1. **Read the table of contents.** This will tell you what is in the book.

2. **Look through the book.** See how the book is set up.

3. **Decide what you want to know.** You may want to refer to one or two sections or review the whole book.

4. **Look in the table of contents.** This will tell you where to find what you are looking for.

5. **Turn to the page listed in the table of contents for the section you want.** Read to find the information you want.

Whole Numbers

- There are 10 symbols: 0,1,2,3,4,5,6,7,8, and 9.
- Each of these number symbols is called a **digit.**
- The number symbols you write to name numbers are called **numerals.**

For example, 36 is called a number, but it is really a **numeral** or a group of **number symbols** which show the number named thirty-six. The number named thirty-six may be shown in the following ways:

36   24 + 12   6 x 6

When a numeral is written in the form 5746, it is written in **standard form**. It can also be written in **expanded form**. This shows how numerals are based on 10.

Standard Form		**Expanded Form**
5746	=	5000 + 700 + 40 + 6
5746	=	5 x 1000 + 7 x 100 + 4 x 10 + 6

Place Value

Each digit has a certain **place value** and **face value.**

- In 5746, the digit 5 is in the thousands place.
- The face value tells us how many ones, tens, hundreds, thousands, etc. there are.
- Let's take 5746 for example.
    - The **face** value of the first digit is **5** and its **place** value is thousands.
    - The **face** value of the second digit is **7** and its **place** value is hundreds.
    - The **face** value of the third digit is **4** and its **place** value is tens.
    - The **face** value of the last digit is **6** and its **place** value is ones.

Remember: when you write a numeral in words, hyphens are used between the tens and ones, so 41 is forty-one.

Let's read the numeral 1527468 and write it in words.

1. Start from the right and mark off groups of three digits. 1/527/468
2. Place commas between each group. 1,527,468
3. Read each group of digits and the name of each group. Start from the left.
4. **1,527,468** reads one million, five hundred and twenty-seven thousand, four hundred and sixty-eight.

Rounding

You do not always need to know the exact number. You can use numbers that are approximate by estimating or **rounding off**.

Remember: It is important to know the place value chart when rounding off numbers.

For example: Let's round 12,837 to the nearest thousand.

1. Underline the number in the place you are rounding off to. 1<u>2</u>, 837
1. Look at the number in the next place to the right. 12,[8]37
2. Eight is greater than 5, so add 1 to the underlined number.
3. Now, change all the numbers to the right of the underlined number to 0.
4. The rounded off number is 13,000 because 12,800 is closer to 13,000 than to 12,000.

Let's try another. Round $538.00 to the nearest hundred dollars. Follow the above steps.

1. $<u>5</u>38.00
2. $5[3]8.00
3. Three is less than 5, so leave the underlined number as it is and don't forget to change all the numbers on the right to 0.
4. The rounded off number is $500.00.

Here's a tricky one to try. Let's round 3,983,542 to the nearest hundred thousand.

1. 3,<u>9</u>83,542
2. 3,9[8]3,542
3. Eight is greater than 5, so add 1 to 9. But when we change the 9 to 10, we can only put the 0 in the hundred thousands place, so we must add the 1 to the 3 to make 4.
4. The rounded off number is 4,000,000.

Try these ones: Round the following to the nearest thousand.

1. 3679	4000
2. 23, 345	23,000
3. 123,456	123,000
4. 128,987	129,000
5. 3187	3000

The Basic Skills

Basic skills are needed to do math questions correctly. These skills are adding, subtracting, multiplying, and dividing. These skills are used with whole numbers, fractions, decimals and percents. You need to know the basic facts really well in order to do the more difficult math problems.

It is important to do math question step by step. Watch for the signs +, -. ×, and ÷ and read and reread the question to know exactly what you are trying to solve.

These symbols are used in many math questions:

- Is less than $<$
- Is greater than $>$
- Is equal to $=$
- Is not equal to $\neq$

Addition

The sign for addition is **+** and it is called a plus sign. This sign tells us to add. The answer is called a **sum** or a **total.**

Let's take 456 + 289.

1. Line up the place values.

$$\begin{array}{r} {\scriptstyle 1\,1\,\,} \\ 456 \\ \underline{+\,289} \\ \mathbf{745} \end{array}$$

2. Add each column starting from the right, adding the ones first. 6 plus 9 is 15. Put the 5 in the ones column and then carry over the 1 to the next column.

3. Add the tens column. 5 plus 8 plus 1 is 14. Put the 4 in the tens column and carry over the one to the next column.

4. Add the hundreds column. 4 plus 2 plus 1 is 7.

5. Your answer is

$$\begin{array}{r} 456 \\ +\,289 \\ \hline 745 \end{array}$$

To check addition problems you can subtract one of the numbers from the sum or answer and you will get the other number.

Estimating Sums

A quick way to estimate the sum of two numbers is to round each number and then add the rounded numbers. This probably won't be the exact answer but it may be close enough for some purposes.

How to estimate a sum by rounding.

1. Round each term that will be added.

2. Add the rounded numbers.

Some uses of rounding are:

- Checking to see if you have enough money to buy what you want.
- Getting a rough idea of the correct answer to a problem

For example: Estimate 235 + 585

1. Round each number. 200 + 600
2. Add. 800
3. Actual answer: 820

How to Improve the Estimate.

1. Round each term that will be added.
2. Add the rounded numbers.
3. If both are rounded down or both rounded up see if the amount of rounding is more than 50. If it is, add or subtract 100 to the estimate.
4. If one number is rounded down and the other is rounded up a closer estimate will not be produced by this method.

For example: Estimate 445 + 735

1. Round each term. 400 + 700
2. Add. 1100
3. Rounded down by more than 50 so add 100. 1100 + 100
4. Rounded answer: 1200
5. Actual answer: 1180

Subtraction

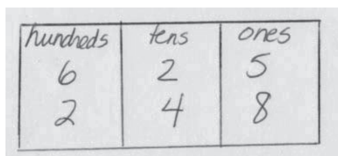
The sign for subtraction is **–** and it is called a minus sign. Subtraction is taking away one number from another and that is why it is called the **minus or take-away** sign.

You can't take a bigger number away from a smaller number so, sometimes you have to regroup numbers to subtract. Regroup means you borrow from the next place value and add it on to a smaller number so then you have a bigger number to subtract the smaller number from.

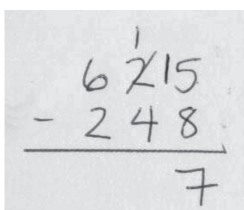
For example: 625 – 248

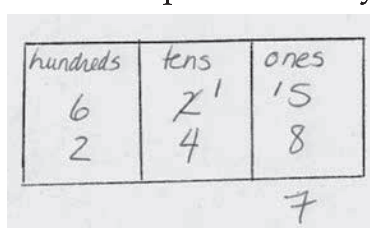
1. Write this problem in a line going down.

$$\begin{array}{r} 625 \\ -\ 248 \\ \hline \end{array}$$

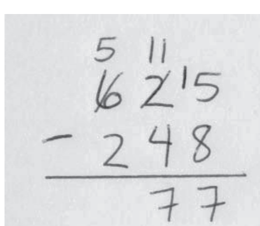


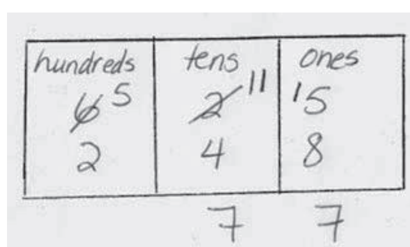
2. Subtract the ones column. You can't take 8 away from 5 so you have to regroup and borrow 1 ten (10) from the tens place. Now you have 15 – 8 = 7



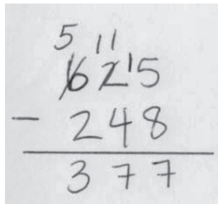


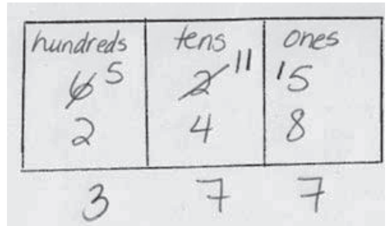
3. Now you subtract the tens column. You borrowed 1 ten so you have 1 ten left. You can't take 4 away from 1 so you have to borrow from the hundreds place. Now you have 11 tens. 11 – 4 = 7





4. You had to borrow 1 hundred from the hundreds place so now you have 5 hundreds left. 5 – 2 = 3 hundreds



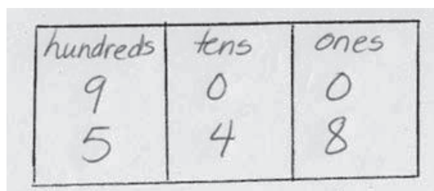


5. 625 – 248 = 377

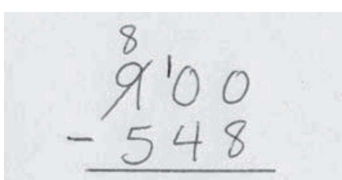
Now let's take 900 – 548

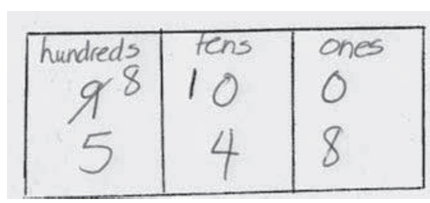
1. Write this problem in a line going down.

$$\begin{array}{r} 900 \\ \underline{-\,548} \end{array}$$

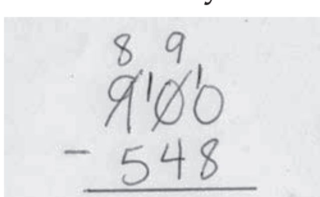


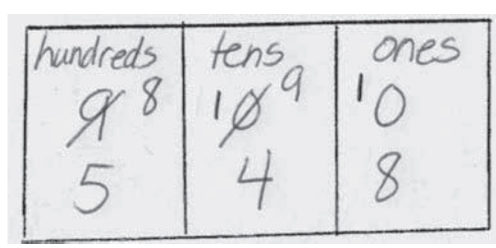
2. Subtract the ones column. You can't take a 8 away from 0 so you have to regroup and borrow 1 ten from the tens place. But there are 0 tens, so you have to borrow 1 hundred from the hundred place.





3. Now you have 10 tens so you can borrow one ten.



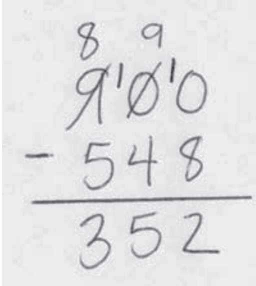


4. Subtract the ones column: 10 – 8 = 2.

   Now subtract the tens column: 9 – 4 = 5

   Now subtract the hundreds column: 8 – 5 = 3

5. 900 – 548 = 352



Remember: Follow the same steps for any subtraction problem**.**

To check subtraction problems you can add the answer to the number you subtracted and you should get the other number.

For example: 352 + 548 = 900

Estimating Differences

We use the same method for estimating differences as we do for adding sums. Round each number and then subtract the rounded numbers. This probably won't be the exact answer but it may be close enough for some purposes.

How to estimate a difference by rounding.

1. Round each term that will be subtracted.

2. Subtract the rounded numbers.

For example: Estimate 855 – 385

1. Round each term.	900 – 400
2. Subtract.	500
3. Actual answer:	470

An estimate can sometimes be improved. If the difference of 645 - 450 were estimated, we would round 645 to 600 and 450 to 500. The estimate would be 600 – 500 or 100. One number was rounded down and the other was rounded up. The number 645 was rounded down by 45 and 450 was rounded up by 50. Adding 45 + 50 gives 95, which rounds to 100. Therefore, a better estimate would be 200. The actual difference is 195.

How can you improve the estimate?

1. Round each term that will be subtracted.
2. Subtract the rounded numbers.
3. If one is rounded down and the other up see if the amount of rounding is more than 50. If it is, add 100 to or subtract 100 from the estimate.
4. If both numbers are rounded down or both are rounded up, a closer estimate will not be produced by this method.

For example: Estimate 955 – 325

1. Round each term.	1000 – 300
2. Subtract.	700
3. Add 55 + 25 = 80 (more than 50) Subtract 100.	700 – 100 = 600
4. Estimation:	600
5. Actual answer:	630

Multiplication

Multiplication is repeated addition. Multiplication is a quicker way to add the same number many times. The sign for multiplication is a times sign **X**. The numbers that are multiplied together are called **factors** and the answer is called the **product.**

For example: 8 x 5 = 40. To get the answer add the number 8 five times.

8 + 8 + 8 + 8 + 8 = 40

Use the same steps to multiply 2, 3, 4, or more digit numbers.

For example: Multiply 355 x 225

1. Arrange in columns. Multiply, starting from the right. Regroup when necessary.

   355
   X 225

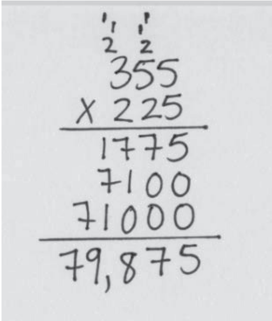
2. Multiply 355 x 5 to get part of the answer 1775 and that is the first partial product. You must regroup – when you multiply 5 x 5 you get 25. Write the 5 down and then carry the 2 to the next column. Then you multiply 5 x 5 again and add the 2 to get 27. Carry the 2 to the next column. Next you multiply 3 x 5 and add 2 to get 17.

3. Place a zero to hold the ones place value before multiplying for the next partial product 355x 20 = 7100 (partial product).

4. Place a zero in the ones and tens place and multiply 355 x 200 = 71000.

5. Add partial products to get the final product.

   355
   X 225
   1775
   7100
   71000
   79,875



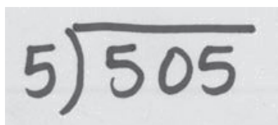
Simple Long Division

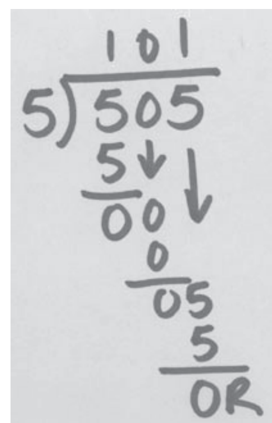
Division is sometimes referred to as the opposite math operation of multiplication.   For example: 40 ÷ 8 = 5. You can reverse this for multiplication: 5 x 8 = 40.

There are two definitions you must know in order to do division:

- The number to be divided into is known as the **dividend** (505 from below).
- The number which divides the other number is known as the **divisor** (5 from below).

For example: 505 ÷ 5





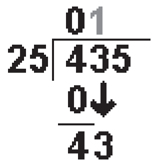
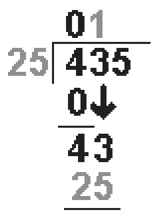
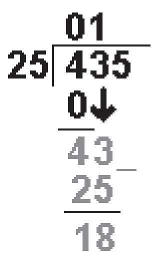
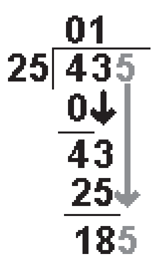
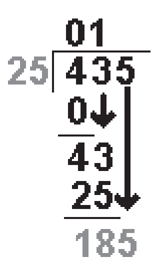
1. How many fives go into 5? 1
2. How many fives go into 0? 0
3. How many fives go into 5? 1
4. Your answer is: 101
5. There is no remainder.

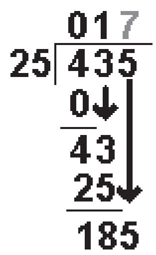
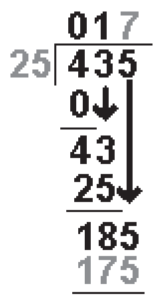
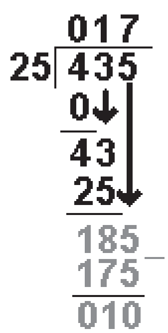
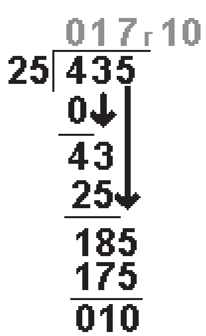
Long Division with Remainders

When we are given a long division to do it will not always work out to a whole number. Sometimes there will be numbers left over. These are known as **remainders**.

For example: 435 ÷ 25

$25\overline{)435}$	4 ÷ 25 = 0 remainder 4	The first number of the dividend is divided by the divisor.
0 $25\overline{)435}$		The whole number result is placed at the top. Any remainders are ignored at this point.
0 $25\overline{)435}$ 0	25 × 0 = 0	The answer from the first operation is multiplied by the divisor. The result is placed under the number divided into.
0 $25\overline{)435}$ 0 4	4 – 0 = 4	Now we **take away** the bottom number from the top number.
0 $25\overline{)435}$ 0↓ 43		Bring down the next number of the dividend.
0 $25\overline{)435}$ 0↓ 43	43 ÷ 25 = 1 remainder 18	Divide this number by the divisor.

		The whole number result is placed at the top. Any remainders are ignored at this point.
	25 × 1 = 25	The answer from the above operation is multiplied by the divisor. The result is placed under the last number divided into.
	43 – 25 = 18	Now we **take away** the bottom number from the top number.
		Bring down the next number of the dividend.
	185 ÷ 25 = 7 remainder 10	Divide this number by the divisor.

		The whole number result is placed at the top. Any remainders are ignored at this point.
	25 × 7 = 175	The answer from the above operation is multiplied by the divisor. The result is placed under the number divided into.
	185 – 175 = 10	Now we **take away** the bottom number from the top number.
		There is still 10 left over but no more numbers to bring down.
		With a long division with remainders the answer is expressed as **17 remainder 10** as shown in the diagram

Properties of Zero:

1. 0 added to any number is the number. $5 + 0 = 5$
2. 0 subtracted from any number is the number. $5 - 0 = 5$
3. The difference between any number and itself is 0. $5 - 5 = 0$
4. When 0 is multiplied by 0, the product is 0. $0 \times 0 = 0$
5. When any other number is multiplied by 0 the product is 0. $5 \times 0 = 0$

Properties of One:

1. Any number multiplied by one is the number. $5 \times 1 = 5$
2. Any number , except 0, divided by itself equals 1. $5 \div 5 = 1$
3. One raised to any power is 1. $1^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

Solving Word Problems

Word problems are math questions in sentence form. Finding the answer is called **solving the problem.**

For example: Lucy Smith earns $1,500 each month. How much does she earn in a year?

1. Read and reread the problem.
2. What does the problem ask you to do?
3. What facts are you given? Sometimes you are given facts that you do not need and sometimes you have to know facts that are not given.
4. Look for clues to help you decide what operation – addition, subtraction, multiplication or division – you need to use.

Addition:

These are word clues that tell you to add.

- **altogether** How much do you have altogether?
- **in all** How many in all?
- **together** Together how much do you have?
- **increase** What increase does that show?
- **total/sum** What is the sum or total amount?

Subtraction:

Subtraction problems ask for **what is left** after something is **taken away.** They may also ask for **how much greater** one number is from another.

Sometimes these words are used:

- **difference** What is the difference?
- **remainder** How many are left over?
- **increase/decrease by**
- **reduce by**
- **less/more** How much less or how much more?
- **smaller** How much smaller?
- **larger** How much larger?
- **farther** How much farther?

Multiplication:

The word clues are often the same for adding and multiplying, because multiplying is just a quick way of adding the same number many times.

The word clues: **total, in all, and altogether,** can mean to add or multiply.

- To find the total of **different** numbers, **add**.
- To find the total of the **same** number many times, **multiply**.

Division:

Dividing is the reverse of multiplying. In multiplying, there are parts and you need a total. In dividing, there is a total and you need to find equal parts of it. A word clue for dividing is each. How much is each or how many in each? Another indicator for division is when you are asked to **find the average.**

Finding Averages

Averages are used in daily living. You talk about the average temperature, average income, or average amount of rain. You find the average by **adding** all the numbers together and then **dividing** that answer by the amount of numbers you added together.

Let's try an example: Find the average temperature for the week.

Monday:	5 degrees C
Tuesday:	7 degrees C
Wednesday:	4 degrees C
Thursday:	9 degrees C

Friday:	12 degrees C
Saturday:	12 degrees C
Sunday:	14 degrees C

To find the average temperature for the week, we add

- 5 + 7 + 4 + 9 + 12 + 12 + 14 = 63
- Now we divide by the number of days 63 ÷ 7 = 9

The average temperature for the week is 9 degrees Celsius.

Order of Operations

To find the answer for certain math questions you need to do more than one operation. Follow the rules below.

Rule 1: First perform any calculations inside brackets.

Rule 2: Next perform all multiplication and division, working from left to right.

Rule 3: Lastly, perform all additions and subtractions, working from left to right.

Let's try 8 x 9 + 24 ÷ 2

You have to perform 3 operations; multiplication, addition and division. This is when you have to know what order they are done in.

1. Multiply 8 x 9 =72 72 + 24 ÷ 2
2. Next, divide 24 ÷ 2 = 12 72 + 12
3. Now, add 72 + 12 = 84

If you don't follow the order of operations, you will get a different answer. That is why everyone must follow this math rule. Some questions have brackets and the question in the bracket needs to be done first.

Let's try 26 + 63 ÷ (15 - 8) x 2 ÷ 3 – 3

1. Brackets first (15 - 8) = 7	26 + 63 ÷ 7 x 2 ÷ 3 – 3
2. Nex,t division 63 ÷ 7 = 9	26 + 9 x 2 ÷ 3 – 3
3. Now, multiply 9 x 2 = 18	26 + 18 ÷ 3 - 3
4. Divide 18 ÷ 3 = 6	26 + 6 - 3
5. Add first 26 + 6	32 – 3
6. And finally subtract	32 – 3 = 29
7. The answer:	29

Factors

Factors are any numbers multiplied together to give a product. What numbers multiply together to get to 4?

- 1 x 4
- 2 x 2

The factors are 1, 2, 4

1 is a **factor** of all numbers because of the **properties** of one.

For example: List the factors of 30:

Ask what two numbers (**factors**) can be multiplied together to make the answer (**product**) 30?

- 1 x 30 = 30
- 2 x 15 = 30
- 3 x 10 = 30
- 5 x 6 = 30

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

Prime Numbers are numbers that have only two factors, one and the number itself.

For example:

- 2 = 1 x 2
- 3 = 1 x 3
- 5 = 1 x 5
- 19 = 1 x 19

Composite Numbers are numbers that have more than two factors.

For example:

- 6 = 1 x 6 and 2 x 3
- The factors of 6 are 1, 2, 3, and 6.

A composite number may be written as a **product** of **prime numbers.**

For example: 12 is a composite number because it has more than 2 factors.

- 12 = 1 x 12 and 2 x 6 and 3 x 4

You can break these factors down to prime numbers.

For example:

- 12 = 2 x 6 (6 = 2 x 3)
- 12 = 2 x 2 x 3 (all prime numbers)

You broke the composite number (12) down to the prime numbers (2x2x3). This shows how a composite number may be written as a product of prime numbers.

Greatest Common Factor is a number that divides evenly into two numbers and it is the largest of all the factors that divides evenly into two numbers. To find the **greatest common factor (G.C.F.)**, you have to list **all** the factors of the two numbers.

For example: Find the G.C.F. of 18 and 24.

1. List all the factors of each.

   a. What two numbers multiplied together make 18?

   b. Start with 1. Ask 1 times what number makes 18? Then go to 2. Ask 2 times what number makes 18? Then 3, and so on.

   c. 18 = 1 x 18, 2 x 9, 3 x 6

   d. What two numbers multiplied together make 24?

   e. 24 = 1 x 24, 2 x 12, 3 x 8, 4 x 6

2. List the factors in order. 18 = 1,2,3,6,9,18 24 = 1,2,3,4,6,8,12,24

3. Find the common factors of the two numbers. 1,2,3,6

4. The greatest (or biggest) of these common numbers is 6.

Multiples

A multiple of a number is the **product or answer** of that number multiplied by any whole number. To find multiples of any number, just multiply that number by 1,2,3,4, and on.

For example: Find the multiples of 6.

1. Multiply 6 by 1,2,3,4, and on.

6x1=6	6x2=12
6x3=18	6x4=24
6x5=30	6 x 6 = 36

2. The multiples of 6 are: 6,12,18,24,30

A number can be a multiple of more than 1 number. These are **common multiples**.

For example: Find the common multiples of 6 and 8.

1. The multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, 48 and on.
2. The multiplies of 8 are: 8, 16, 24, 32, 40, 48, 56, 64 and on.
3. The common multiples of 6 and 8 are 24 and 48. The least common multiple is 24.

Fractions

A **fraction** is a **part** of a **whole.** A penny is a fraction of a dollar. It is 1 of the 100 **equal parts** of a dollar or 1/100 (one hundredth) of a dollar. Five days are a fraction of a week. They are 5 of the 7 equal parts of a week or 5/7 of a week

Fractions describe part of a number. Fractions have two parts:

- **Numerator** - tells how many parts you have.
- **Denominator** - tells how many parts in the whole.

Think of **nu** in numerator as the "Nu" for the **number up** and **d** in denominator as the "D" for the **number down**. The line between means division.

Fractions are parts or divisions of a whole. The denominator divides the numerator.

Note: Fractions are shown in this form ½ and sometimes this way 1:2

Equivalent Fractions

Equivalent fractions are fractions that:

- Are equivalent or equal to each other.
- Have the same value.
- Look different.
- May be in lower terms or higher terms.

For example:

- 2 is half of 4 and 1 is half of 2 and 50 is half of 100.

$\frac{2}{4}$ $\frac{1}{2}$ $\frac{50}{100}$

- All these fractions are equal to 1/2 (one-half) because they describe the same thing in different ways. They are equivalent fractions.

To find if fractions are equivalent, you cross-multiply.

1. Take the first numerator and multiply it by the second denominator.
2. Next take the first denominator and multiply it by the second numerator.

For example: Are $\frac{1}{3}$ and $\frac{4}{12}$ equivalent fractions?

- 1 x 12= 12 3 x 4= 12 Yes, they are equivalent fractions.

To rename equivalent fractions you multiply or divide both the numerator and denominator by the same number. This does not change the value of the fraction because you are multiplying or dividing by 1. 1 can be shown as 1/1, 2/2, 3/3, 4/4 and on.

For example: Write 3 fractions equivalent to $\frac{1}{3}$.

- $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$
- $\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$
- $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$

For example: Write 3 different fractional names for 1.

- $1 = \frac{1}{1}$
- $1 = \frac{2}{2}$
- $1 = \frac{3}{3}$
- 1 can be $\frac{2}{2}, \frac{3}{3}, \frac{6}{6}, \frac{10}{10}$

1 can be 6/6, because the bottom number (or the denominator) 6 tells us that the whole is divided into 6 parts. The top number (or the numerator) 6 tells us how many parts are there. If you divide a whole into 6 parts and all 6 parts are there, then you still have one whole or 1. 6/6 = 1

Renaming Equivalent Fractions

To rename an equivalent fraction is to change the form (the numbers) without changing the value. You can raise **fractions to higher terms.**

For example: Write the fraction 2/5 in higher terms. Remember you can multiply by any form of one.

$$\frac{2}{5} \times 1 = \frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$$

$$\frac{2}{5} \times \frac{4}{4} = \frac{8}{20}$$

2/5 and 6/15 and 8/20 are all equivalent fractions.

Check to see if the above are equivalent fractions. **Remember:** Cross-multiply

- $\frac{2}{5} = \frac{6}{15}$ 2 x 15 = 30 5 x 6 = 30
- $\frac{2}{5} = \frac{8}{20}$ 2 x 20 = 40 5 x 8 = 40

Sometimes you are asked to complete a fraction in higher terms. Then you are given just part of a value.

For example: Complete in higher terms. $\frac{3}{4} = \frac{?}{16}$

1. Ask what was done to the 4 to equal 16? It was multiplied by 4.
2. Always do the same to both terms. If you multiply the denominator by 4, you must multiply the numerator by 4.
3. Now multiply the 3 by 4 to get 12. $\frac{3\ (\times 4)}{4\ (\times 4)} = \frac{12}{16}$
4. Cross-multiply to see if this is an equivalent fraction. 3 x 16 = 48
   4 x 12 =48
5. They are equivalent.

You can rename an equivalent **fraction to lowest terms**. It is called simplifying the fraction or reducing the fraction.

For example: Write this fraction 6/8 in its simplest form.

- factors of 6 = 1,2,3,6
- factors of 8 = 1,2,4,8

Greatest common factor = 2

Remember: divide both the denominator and numerator by 2.

$$\frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$$

Let's try some more:

1. 12/15 factors of 12 = 1, 2, 3, 4, 6, 12 factors of 15 = 1, 3, 5, 15

   Greatest common factor = 3

   $$\frac{12}{15} \div \frac{3}{3} = \frac{4}{5} \text{ (lowest term)}$$

2. 25/30 factors of 25 = 1, 5, 25 factors of 30 = 1, 2, 3, 5, 6, 10, 15, 30

   Greatest common factor = 5

   $$\frac{25}{30} \div \frac{5}{5} = \frac{5}{6} \text{ (lowest term)}$$

Adding Fractions

To add fractions with the same denominator, add only the numerators and put the sum over the denominator.

For example: Add 2/8 + 3/8

$$\frac{2+3}{8} = \frac{5}{8}$$

Try this one: Add 1/6 + 2/6 (Write answer in simplest form.)

$$\frac{1+2}{6} = \frac{3}{6} \qquad \frac{3}{6} \div \frac{3}{3} = \frac{1}{2} \text{ (simplest form)}$$

Subtracting Fractions

To subtract fractions with the same denominator, subtract only the numerators and put the difference over the denominator.

For example: Subtract 5/10 - 3/10 (Write answer in simplest form.)

$$\frac{5-3}{10} = \frac{2}{10} \qquad \frac{2}{10} \div \frac{2}{2} = \frac{1}{5} \text{ (simplest form)}$$

Proper Fractions

Proper fractions are fractions that are less than 1.

- 1/3, 3/4, 4/7, and 5/9 are all examples of proper fractions

The numerator is smaller than the denominator.

Improper Fractions

Improper fractions are fractions that are equal to or greater than 1.

- 2/2, 4/4, 6/5 and 12/7 are examples of improper fractions

The numerator of an improper fraction is the same as or larger than the denominator.

Mixed Numbers

A mixed number is made up of a whole number and a fraction.

$1\frac{1}{2}, 3\frac{2}{3}, 12\frac{1}{4}$ are examples of mixed fractions

Change a Mixed Number to an Improper Fraction

A mixed number can be renamed as an improper fraction. **For example:** $1\frac{1}{2}$

1. 1 and 1/2 is a mixed number
2. 1 is the whole number
3. 1/2 is the fraction
4. Change 1 to 2/2
5. Add $\frac{2}{2} + \frac{1}{2} = \frac{3}{2}$

There is a quicker way to change mixed numbers into improper fractions.

Try this one: Write this mixed number as an improper fraction. $3\frac{2}{5}$

1. Take the whole number (3) and multiply it by the denominator of the fraction (5)
2. 3 x 5 = 15
3. Then add the numerator of the fraction (2) to that number
4. 15 + 2 = 17
5. Your improper fraction is $\frac{17}{5}$

Change an Improper Fraction to a Mixed Number or Whole Number

An improper fraction can be renamed as a whole number or a mixed number.

For example: 35/7

Remember 35 is the numerator that tells the number of equal parts and **7** is the denominator that tells the number of equal parts into which the whole is divided.

1. 35 ÷ 7 = 5 with no remainder
2. The whole number is 5

Try this one: 21/6

1. 21 ÷ 6 = 3 with 3 remainder
2. 3 is the whole number
3. 3/6 is the remainder
4. 3 and 3/6 is the mixed number – however we can simplify the 3/6
5. $\frac{3}{6} \div \frac{3}{3} = \frac{1}{2}$
6. 21/6 can be renamed as the mixed number 3 ½

Adding Fractions When the Answer is an Improper Fraction

When fractions are **added** and the answer is an improper fraction, the improper fraction should be **renamed as a whole number or a mixed number.**

For example: $\frac{3}{4} + \frac{1}{4} = \frac{4}{4}$ $\frac{4}{4}$ equals 1

Try this one: 5/9 + 7/9

1. $\frac{5+7}{9} = \frac{12}{9}$
2. 12 ÷ 9 = 1 plus 3 remainder
3. 1 and 3/9 (need to simplify)
4. $\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$
5. Answer is 1 ⅓

Adding Mixed Numbers

When mixed numbers are added, answers need to be renamed and put in the simplest form.

For example: $5\frac{7}{12} + 8\frac{8}{12} + 1\frac{3}{12}$

1. First add all the whole numbers. $5 + 8 + 1 = 14$
2. Next add all the numerators. $\frac{7+8+3}{12} = \frac{18}{12}$
3. Since $\frac{18}{12}$ is more than 1 you must convert to a mixed number.
4. $18 \div 12 = 1$ with 6 remainder 1 and $\frac{6}{12}$
5. Add the whole numbers together. $14 + 1 = 15$
6. Answer is 15 and 6/12. You must now simplify.
7. $\frac{6}{12} \div \frac{6}{6} = \frac{1}{2}$
8. The answer is 15 ½.

Subtracting Mixed Numbers

In order to **subtract** a mixed number from a whole number, the whole number needs to be renamed as a mixed number. Make the denominator of its fraction the same as the denominator of the other fraction.

For example: $5 - 2\frac{4}{5}$

1. Change the whole number 5 to a mixed number.
2. 5 can be changed to 4 and $\frac{5}{5}$.
3. Subtract the whole numbers and then subtract the fractions.

   $4\frac{5}{5} - 2\frac{4}{5} = 2\frac{1}{5}$

4. The answer is $2\frac{1}{5}$

Unlike Fractions

Unlike fractions do not have the same denominator. To compare, add, or subtract these unlike fractions, they have to be renamed with the least common denominator. To find the least common denominator of two fractions, list the multiples of each fraction.

For example: Find the least common denominator for 1/4 and 5/6

1. Multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36…
2. Multiples of 6 = 6, 12, 18, 24, 30, 36…
3. Common multiples = 12, 24, 36.
4. The least common multiple is 12 therefore the common denominator is 12.
5. Rename the fractions changing both denominators to 12.

6. Ask what was done to the 4 (denominator) to make 12? It was multiplied by 3, so multiply the numerator by 3. (1 x 3 = 3)

7. Ask what was done to the 6 (denominator) to make 12? It was multiplied by 2, so multiply the numerator by 2. (5 x 2 = 10)

$$\frac{1\ (\times 3)}{4\ (\times 3)} = \frac{3}{12} \qquad \frac{5\ (\times 2)}{6\ (\times 2)} = \frac{10}{12}$$

8. The fractions are changed to $\frac{3}{12}$ and $\frac{10}{12}$. We can now add or subtract them.

Adding and Subtracting Unlike Fractions

To add or subtract unlike fractions, rename them with the same denominator. Follow these steps:

1. List the multiples for each denominator and find the least common denominator.
2. Rename the denominators and multiply the numerators.
3. Add or subtract the fractions and then the whole numbers.
4. Write the answer in the simplest form.

Adding:

Try this one: $4\frac{1}{2} + 2\frac{3}{8}$

1. Multiples of 2 = 2, 4, 8, 12, 16, 20, 24, 28, 32, 3…
2. Multiples of 8 = 8, 16, 24, 32, 40, 48…
3. Least common multiple is 8
4. Multiply $\frac{1}{2}$ by $\frac{4}{4}$ to get a denominator of 8. $\frac{1\ (\times 4)}{2(\times 4)} = \frac{4}{8}$

5. 3/8 stays the same.
6. Add together: $4\frac{4}{8} + 2\ \frac{2}{3}\ = 6\frac{7}{8}$

Subtracting:

Try this one: $\frac{3}{5} - \ \frac{4}{15}$

1. Multiples of 5 = 5, 10, 15, 20, 25…
2. Multiples of 15 = 15, 30, 45, 60…
3. Least common multiple is 15
4. Multiply $\frac{3}{5}$ by $\frac{3}{3}$ to get a denominator of 15. $\frac{3(\times 3)}{5(\times 3)} = \frac{9}{15}$
5. $\frac{4}{15}$ (stays the same)
6. Now subtract $\frac{9}{15} - \frac{4}{15} = \frac{5}{15}$
7. Now reduce to simplest form. You must divide both numerator and denominator by 5. $\frac{5}{15} \div \frac{5}{5} = \frac{1}{3}$
8. Answer is: $\frac{1}{3}$

Sometimes mixed numbers have to be regrouped before they can be subtracted.

Multiplying of Fractions

Multiplying fractions is very different from adding or subtracting. It takes lots of practice to remember the different steps.

1. Multiply the numerators.
2. Multiply the denominators.
3. Write the answer in simplest form.

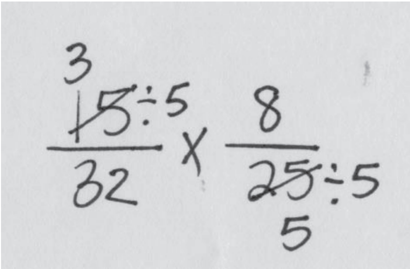
For example: $\frac{3}{5} \times \frac{4}{6}$

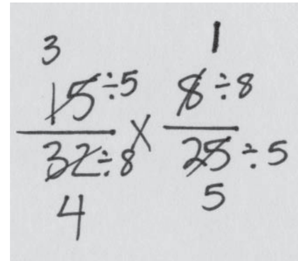
1. Multiply the numerators. $3 \text{ x } 4 = 12$
2. Multiply the denominators. $5 \text{ x } 6 = 30$
3. Answer: $\frac{12}{30}$
4. Now reduce to simplest form. You must divide both numerator and denominator by 6. $\frac{12}{30} \div \frac{6}{6} = \frac{2}{5}$

Sometimes when fractions are multiplied, a shortcut is used. Especially when you are dealing with really large numbers it is much easier to use the short-cut method.

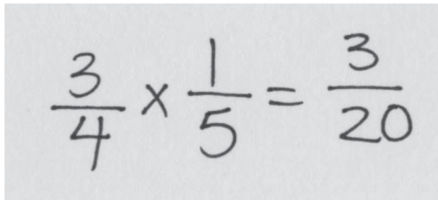
For example: Multiply $\frac{15}{32} \times \frac{8}{25}$

1. Ask yourself if you can reduce the numerator and denominator by dividing.
2. 5 will divide into both 15 and 25.





3. 8 will divide into both 8 and 32.
4. Now you can multiply the numbers that are left.



5. Your answer is: $\frac{3}{20}$

Multiplying Fractions and Whole Numbers

Follow these steps to multiply fractions and whole numbers:

1. Rename the whole number as a fraction with a denominator of 1.
2. Use the shortcut.
3. Multiply the numerators and then multiply the denominators.

For example: 3 x $\frac{3}{5}$

1. Rename the whole number to $\frac{3}{1}$.
2. Use the shortcut – can't cross anything out.
3. Multiply. $\frac{3}{1} \times \frac{3}{5} = \frac{9}{5}$
4. Change to mixed number. 1 ⅘

Multiplying Fractions and Mixed Numbers

When multiplying fractions and mixed numbers rename the mixed number as an improper fraction and multiply as you would any fractions.

For example: $2\frac{1}{4} \times 3\frac{1}{5}$

1. Change mixed numbers to improper fractions. Remember that you can multiply the whole number by the denominator and then add the numerator.

   a. $2\,{}^{1}\!/\!_{4} = \frac{9}{4}$

   b. $3\,{}^{1}\!/\!_{5} = \frac{16}{5}$

2. Cross out what you can: $\frac{9}{\cancel{4}_{1}} \times \frac{\cancel{16}^{4}}{5}$ (4 goes into 16 four times)

3. Now multiply $\frac{9}{1} \times \frac{4}{5} = \frac{36}{5}$

4. $\frac{36}{5}$ is your answer - now put it into a mixed number.

5. 36 ÷ 5 = 7 with 1 left over.

6. $7\frac{1}{5}$ is your final answer.

Dividing Fractions

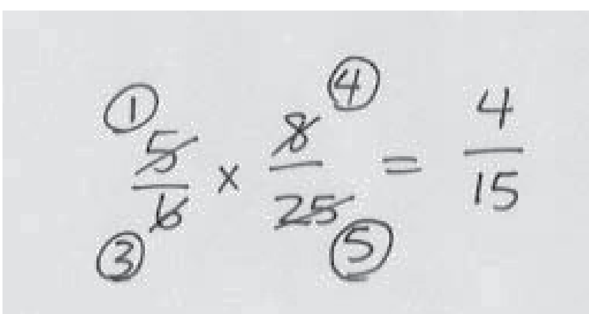
Before you learn to divide fractions, you need to understand the term **reciprocal.** Reciprocal means flip the fraction. So the reciprocal of 1/2 would be 2/1.

To solve any division problem that has a fraction or mixed number in it:

1. Change any mixed number or whole number to an improper fraction.
2. Turn the fraction you are dividing by (the second fraction) upside down (**reciprocal**) and change the division sign to multiplication.
3. Use the shortcut if possible, multiply, and reduce answer to lowest terms.

For example: $\frac{5}{6} \div 3\frac{1}{8}$

1. First change $3\frac{1}{8}$ to an improper fraction. $\frac{25}{8}$
2. Next find the reciprocal of $\frac{25}{8}$. $\frac{8}{25}$
3. Now multiply (reduce if you can). $\frac{5}{6} \times \frac{8}{25}$



4. Your answer is: $\frac{4}{15}$

Decimals

Decimals, like fractions, allow you to work with **parts** of **numbers.** Decimals are names for fractions. If a fraction can be written with a denominator of 10, 100, 1000 and on, a decimal can be used to name the number.

Decimals have different **place values**. They are places to the **right of the decimal point.** You have been working with decimals for a long time. In money, any amount less than a dollar is a decimal.

Money	**Decimal Part of a Dollar**	**Fractional Part of a Dollar**
1 cent	$.01	1/100
10 cents	$.10	10/100
25 cents	$.25	25/100
50 cents	$.50	50/100

Reading Decimals

- The **dot** in the decimal is called the **decimal point.**
- It is written to the right of the ones place.
- The first place to the right after the decimal point is the tenths place.
- Place value continues on both sides of the decimal point.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths	Millionths
		6	5	2	1	8	.	3	4	7	9		

- The number written under the chart is 65,218.3479.
- It is read "sixty-five thousand, two hundred eighteen and three thousand four hundred seventy-nine thousandths."
- The comma helps you read whole numbers by marking off the periods (thousands, millions, billions).
- The decimal point is different. It shows you where the whole number ends and the decimal number begins.

- The decimal point does not take up a place. Only a number can take up a place.
- Any decimal no matter how large is less than the whole number 1.
- Any number that has both a whole number and a decimal number in it is larger than a number that has a decimal only.

Comparing Decimals

If there are no whole numbers, or if the whole numbers are the same, you have to compare the decimal number. Here is a trick you can use to compare decimals.

For example: Which is larger? 0.07 or 0.2?

1. **Add one zero at the end of** 0.2 – by adding the zero, you have 0.07 and 0.20
2. Compare: 20 hundredths is larger than 7 hundredths
3. The answer is **0.2**

Let's try this one: Arrange the following decimals in order from the smallest to the largest: 0.8, 0.08, 0.088, 0.808.

1. **Add zeros** so each decimal has three places. .800, .080, .088, .808
2. Compare and arrange the decimals in the correct order: .080, .088, .800, .808
3. Leave the zeros out in the final answer. **.08, .088, .8, .808.**

Rounding Decimals

Decimals are rounded the same way whole numbers are rounded.

For example: Round 9.635 to the nearest one.

1. The number to the right of the ones is 6.
2. 6 is greater than 5.
3. The ones number is rounded up one.
4. 9.635 rounded to the nearest one is 10.

Let's try this one: Round 9.635 to the nearest tenth.

1. The number to the right of the tenths is 3.
2. 3 is less than 5.
3. The tenths number stays the same.
4. 9.635 rounded to the nearest tenth is 9.6.

Let's try one more: Round 9.635 to the nearest hundredth.

1. The number to the right of the hundredths is 5.
2. 5 is 5 or greater.
3. The hundredths number is rounded up one.
4. 9.635 rounded to the nearest hundredth is 9.64.

Changing Decimals to Fractions

You can change a decimal to a fraction or a mixed number.

1. Read the decimal.
2. Write it as a fraction with a denominator of 10, 100, 1000, etc.
3. Rewrite the fraction in simplest form.

For example:

- 0.8 = $\frac{8}{10} \div \frac{2}{2} = \frac{4}{5}$
- 0.95 = $\frac{95}{100} \div \frac{5}{5} = \frac{19}{20}$
- 0.015 = $\frac{5}{1000} \div \frac{5}{5} = \frac{3}{20}$

Decimals can be renamed or changed to mixed numbers. To rename or change decimals to mixed numbers use the above steps.

For example:

- 2.25 is 2 $\frac{25}{100}$ (change to simplest form)
- $\frac{25}{100} \div \frac{25}{25} = \frac{1}{4}$
- 2.25 = 2 $\frac{1}{4}$

Changing Fractions to Decimals

Fractions or mixed numbers with denominators of 10, 100, 1000, etc. can be written as decimals.

Remember: Put in the place value. The first place to the right of the decimal point is the tenths place. The second place to the right of the decimal point is the hundredths place. The third place to the right of the decimal point is the thousandths place and so on.

For example:

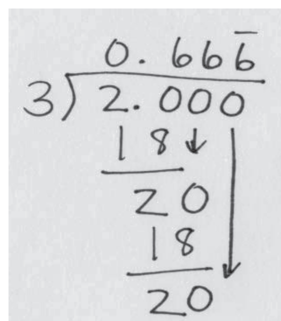
- $\frac{47}{100}$ = 0.47
- $\frac{5}{10}$ = 0.5
- 1 and $\frac{9}{100}$ = 1.09
- 3 and $\frac{20}{100}$ = 3.2
- $\frac{7}{100}$ = .07
- $\frac{8}{1000}$ = .008
- $\frac{88}{1000}$ = .088
- $\frac{88}{100}$ = .88
- 1 and $\frac{8}{100}$ = 1.08
- 1 and $\frac{8}{1000}$ = 1.008

There is another way to rename a fraction to a decimal. If a fraction does not have a denominator of 10.100, 1000, etc. you can divide the bottom number into the top number.

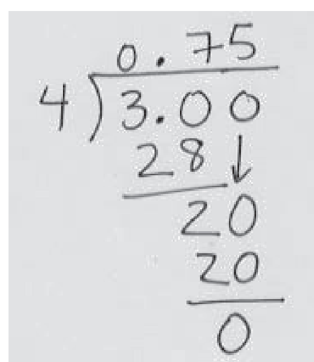
Follow the steps below:

1. Divide the bottom number into the top number.
2. Add a decimal point and zeros. Divide and bring the decimal point up.
3. You may get a repeating decimal. You can repeat the decimal three time and then add a dot above the decimal to indicate it repeats itself.

For example: Change ⅔ to a decimal.



Try this one: Change ¾ to a decimal.

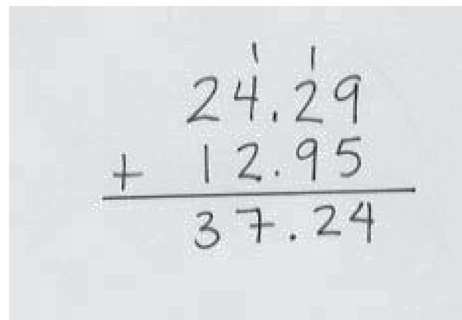


Adding Decimals

Decimals are added the same way whole numbers are added. To add decimals, line up the decimal points and add as you would whole numbers.

For example: Add 24.29 + 12.95.

1. Write the question so the decimal points are lined up.
2. Add the hundredths.
3. Add the tenths.
4. Add the whole numbers.
5. Line up the decimal points.



Remember : Any whole number is understood to have a decimal point at its right. You can add zeros to the right so it is easier to add.

For example: 16 + 2.005.

16.000
2.005
18.005

Estimating the Sum of Two Decimals

How do you estimate a sum by rounding?

1. Round each decimal term that will be added.
2. Add the rounded terms.

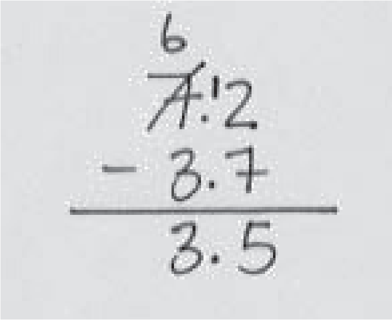
For example: Estimate the sum of 0.988 + 0.53

1. Round each number. 1 + 0.5
2. Add the rounded numbers. 1.5
3. The actual sum is: 1.518

Subtracting Decimals

Decimals are subtracted the same way whole numbers are subtracted. You just have to make sure the decimal points are lined up.

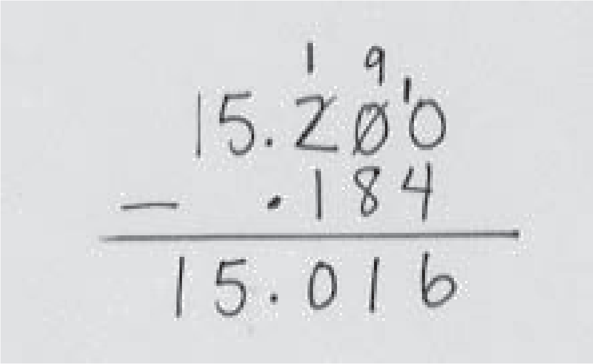
For example: Subtract 7.2 – 3.7



1. Write the question so the decimal points are lined up.
2. Subtract the tenths.
3. Subtract the whole numbers.
4. Line up the decimal points.

Remember: To subtract put the larger number on top and you can add as many zeros after the decimal point as you need.

For example: Subtract 15.2 –0.184



Follow the above steps, but add zeros to give the top number the same number of places as the bottom number.

Estimating the Difference of Two Decimals

How do you estimate a difference by rounding?

1. Round each decimal term that will be subtracted.
2. Subtract the rounded terms.

For example: Estimate the difference of 0.988 – 0.53

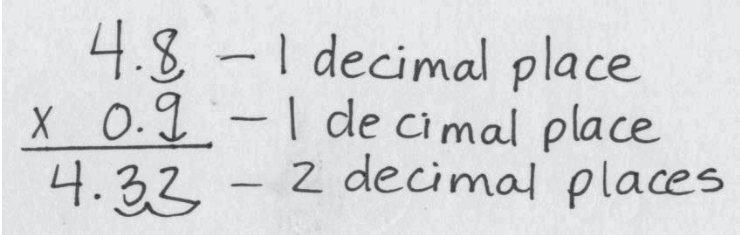
1. Round. 1 – 0.5
2. Subtract the rounded numbers. 0.5
3. The actual difference is: 0.458

Multiplying Decimals

To multiply decimals, multiply the same way you would multiply whole numbers, except you must put the decimal point in. Count the number of decimal places in both numbers you are multiplying and put the total number of places in your answer.

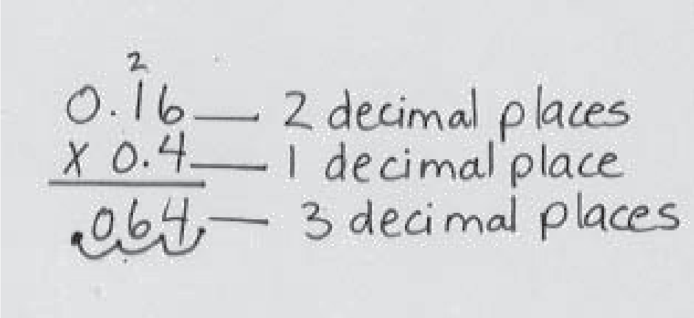
For example: Multiply 4.8 x 0.9

1. Arrange in columns and multiply.
2. Place the decimal point in the answer so that the number of places in the answer is the same as the two numbers that were multiplied.



Sometimes it is necessary to write extra zeros in the answer before the decimal point can be placed.

For example: 0.16 x 0.4



Multiplying Decimals by 10, 100, and 1000

There are shortcuts when multiplying decimals by 10, 100 and 1000.

- Move the decimal point one place to the right when you multiply by 10.
  0.34 x 10 = 3.4
- Move the decimal point two places to the right when you multiply by 100.
  0.34 x 100 = 34
- Move the decimal point three places to the right when you multiply by 1000.
  0.34 x1000 = 340

Notice the pattern for multiplying a number by 0.1, 0.01, or 0.001.

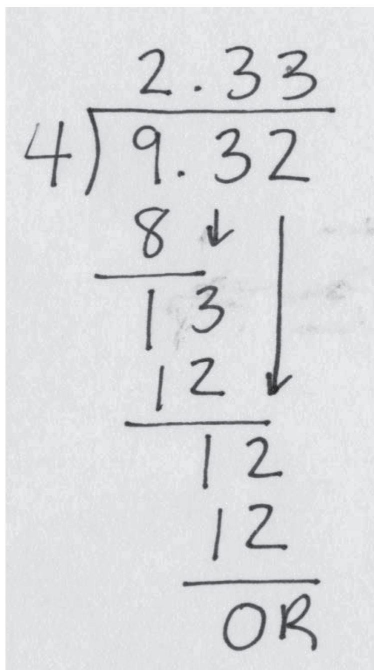
- Move the decimal point one place to the left when you multiply by 0.1.
  637 x 0.1 = 63.7
- Move the decimal point two places to the left when you multiply by 0.01.
  637 x 0.01 = 6.37
- Move the decimal point three places to the left when you multiply by 0.001.
  637 x 0.001 = .637

Dividing Decimals

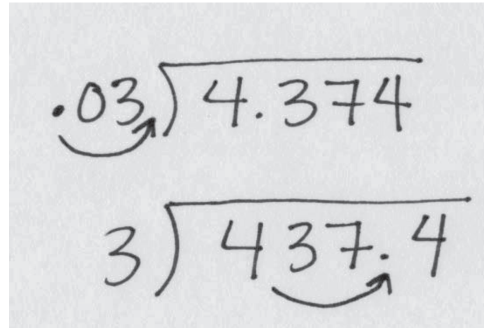
The division steps for decimals are the same as they are for whole numbers, except you must place the decimal point in the answer.

Divide a decimal by a whole number.

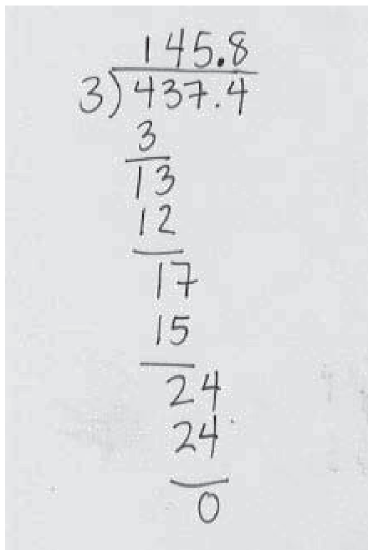
To divide a decimal by a whole number, bring the decimal point up in the answer directly above the decimal point in the question.



Divide a decimal by a decimal.



1. Move the decimal point in the divisor to the right as far as it will go
2. Move the point in the dividend the same number of places.
3. Bring the point up in the answer directly above its new place and divide.



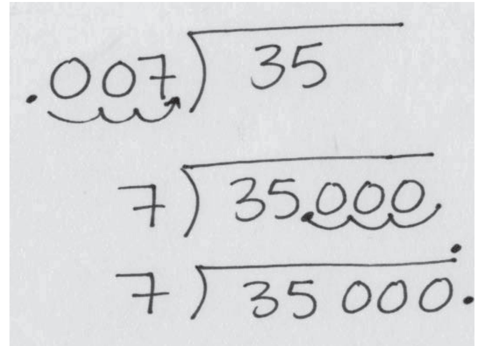
To divide a decimal by a decimal, change the problem to one in which you are dividing by a whole number.

Divide a whole number by a decimal.

1. Move the point in the divisor three places to the right.

2. Place a point to the right of the whole number and move it three places to the right, holding each place with a zero.

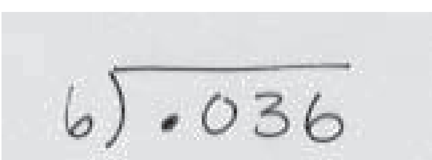
3. Bring the decimal point up in the answer and divide.

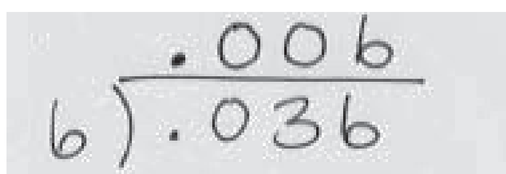


When dividing a decimal into a whole number, put a decimal point after the whole number and in order to move the point enough places add zeros to hold the place.

Remember: It is understood that a whole number has a decimal point at its right. Sometimes when decimals are divided by whole numbers, zeros have to be put in the answer to hold a place**.**

For example:





In this example, zeros are put in the answer to show there is no tenths or hundredths in the answer.

Dividing Decimals by 10, 100, and 1000

Like shortcuts in multiplying decimals by 10, 100, or 1000, there are also shortcuts in dividing decimals by 10, 100, and 1000.

1. When multiplying decimals by 10, for example, move the decimal point one place to the right and the number gets bigger.
2. When dividing decimals by 10, move the decimal point one place to the left and the number gets smaller. When dividing by 100 move the decimal point two places to the left. When dividing by 1000 move the decimal point three places to the left.
3. If multiplying by a decimal, for example 52 x 0.1 = 5.2, the decimal moves one place to the left.
4. If dividing by a decimal, for example: 52 ÷ 0.1 = 520, the decimal moves one place to the right.
5. Notice the pattern in the chart on the next page.

Multiply

by 10	by 100	by 1000
3.65 x 10 = 36.5	3.65 x 100 = 365	3.65 x 1000 = 3650
0.584 x 10 = 5.84	0.584 x 100 = 58.4	0.584 x 1000 = 584

by .1	by .01	by .001
189 x .1 = 18.9	189 x .01 = 1.89	189 x .001 = .189
1.72 x .1 = .172	1.72 x .01 = .0172	1.72 x .001 = .00172

Divide

by 10	by 100	by 1000
25.9 ÷ 10 = 2.59	25.9 ÷ 100 = .259	25.9 ÷ 1000 = .0259
13 ÷ 10 = 1.3	13 ÷ 100 = .13	13 ÷ 1000 = .013

by 10	by 100	by 1000
.42 ÷ .1 = 4.2	.42 ÷ 100 = 42	.42 ÷ 1000 = 420
19 ÷ .1 = 190	19 ÷ .01 = 1900	19 ÷ .001 = 19000

Percents

Percent is another way to describe a **part** or **fraction** of something. The only denominator that a percent can have is 100. This denominator is shown by a **percent sign %.** The word percent and the sign % both mean hundredths.

For example: 25% is the same as 25/100 or 1/4 or .25

Although percent means out of one hundred, a percent can be more than 100. A percent larger than 100 is equal to an improper fraction. When working with percent problems, you need to change the percent to a decimal or fraction.

Changing Percents to Decimals

Here is how to change 85% to a decimal:

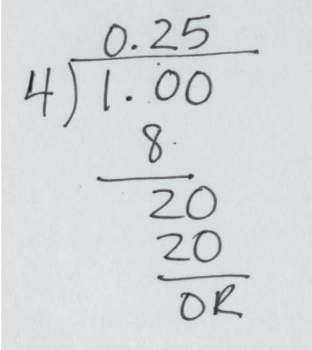
1. Drop the percent sign. 85
2. In a number with no decimal point, it is understood that the decimal point is at the end of the number. Put in the decimal point. 85.
3. Now that you have a decimal point, move it two places to the left. 0.85 = **85% = .85 = 85 hundredths**

The tricky part is putting in the decimal point. Study these examples:

- 5 % = .05
- 10% = .10
- 55% = .55
- 3% = .03

For example: Change 25 1/4 % to a decimal.

1. Drop the percent. $25\frac{1}{4}$
2. Change 1/4 to a decimal. 4 ÷ 1= .25 →
3. Put in the decimal point. 25.25
4. Move the decimal point two places to the left. 0.2525



The next example will show that when a percent like 100%, 200%, 300% etc. is changed to a decimal, the answer is a whole number.

Here's another example: Change 100% to a decimal.

1. Drop the %. 100
2. Put in the decimal point. 100.
3. Move the decimal point two places to the left. 1.00 =1

100% = 1

Changing Decimals to Percents

You just learned how to change a percent to a decimal by moving the decimal point two places to the left. Changing a decimal to a percent is just the opposite.

For example: Change 0.43 to a percent.

1. Move the decimal point two places to the right.	0.43
2. The decimal point is at the end of the number where it can be dropped.	43
3. Add the percent.	43%

Try this one. Change 0.217 to a percent.

1. Move the decimal point two places to the right.	21.7
2. The decimal point is not at the end of the number so you cannot drop it.	21.7
3. Add the percent.	21.7%

Changing Percents to Fractions

Now that you can change a percent to a decimal, you can change a percent to a fraction because you can use that as the first step in this example.

For example: Change 15% to a fraction.

1. Change the percent to a decimal. 0.15
2. Now change the decimal to a fraction. $0.15 = \frac{15}{100}$
3. Reduce the fraction. $\frac{15}{100} \div \frac{5}{5} = \frac{3}{20}$

There is a shorter way to change a percent to a fraction. The only denominator a percent can have is 100.

1. Use the number in front of the % as a numerator. 15% = 15
2. Always use 100 as the fractions denominator. $\frac{15}{100}$
3. Reduce the fraction. $\frac{15}{100} \div \frac{5}{5} = \frac{3}{20}$

It is good to know both methods because the next example needs to be changed to a decimal first before you can change the percent to a fraction.

Here's another example: Change 37.5 % to a fraction.

1. Change the percent to a decimal. .375
2. Change the decimal to a fraction. $\frac{375}{1000}$
3. Reduce. $\frac{375}{1000} \div \frac{125}{125} = \frac{3}{8}$

This example takes an extra step.

One more example: Change 6 1/4% to a fraction.

1. Change the fraction to a decimal. ¼ = 0.25
2. Put 6.25 over 100. $\frac{6.25}{100}$
3. Reduce (divide both by 6.25). $\frac{1}{16}$

Percents and their Values as Fractions

These are some common fractions and percents. It is helpful to know what each of them is equal to.

Fraction	Percent	Fraction	Percent
1/2	50%	1/8	12.5 %
1/4	25%	3/8	37.5%
3/4	75%	5/8	62.5%
1/5	20%	7/8	87%
2/5	40%	1/10	10%
3/5	60%	3/10	30%
4/5	80%	7/10	70%
1/3	33.33%	9/10	90%
2/3	66.66%	1/6	16.66%
		5/6	83.33%

Changing Fractions to Percents

There are two ways to change a fraction to a percent.

Method 1:

Multiply the fraction by 100%.

$$\frac{3}{\cancel{4}_{1}} \times \frac{\overset{25}{\cancel{100\%}}}{1} = \frac{75\%}{1} = 75\%$$

Method 2:

Divide the bottom number of the fraction into the top number and then multiply by 100 (move the decimal place to the right 2 times)

$$\begin{array}{r} 0.75 \\ 4\overline{)\,3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

.75 x 100 = 75%

Answer: ¾ = 75%

Finding a Percent of a Number

Percents are very common in the everyday world. Learning how to solve percent problems will be helpful in figuring out discounts on items you buy or figuring interest on loans. There are different methods for finding a percent of a number.

For example: Find 10% of 150.

1. Change the percent to a decimal. 10% = 0.10
2. Multiply. 0.10 x 150 = 15
3. 10% of 150 is 15.

Let's try this one: Find 7% of 40.

1. Change the percent to a decimal. 7% = 0.07
2. Multiply. 0.07 x 40 = 2.8
3. 7% of 40 is: 2.8

Finding what percent one number is of another is a similar problem that can be solved by writing the problem as a proportion. Notice the difference in the way this problem is written as a proportion.

Follow this example: What percent of 45 is 9?

1. Write the unknown percent as a fraction, using x to stand for the unknown. X% = x/100.

2. Use the fraction to write the problem as a proportion.

$$\frac{x}{100} = \frac{9}{45}$$

3. Cross multiply.

$$45x = 900$$
$$x = \frac{900}{45}$$
$$x = 20\%$$

4. 9 is 20% of 45

Let's try this one: 48 is 16% of what number?

1. Write the percent as a fraction. 16% =16/100.

2. Use the fraction to write the problem as a proportion.

$$\frac{16}{100} = \frac{48}{x}$$

3. Cross multiply.

$$16x = 4800$$
$$x = 4800 \div 16$$
$$x = 300$$

4. 48 is 16% of 300.

Calculating Interest

Interest is the money you pay for borrowing money. The amount of interest you pay on any loan depends on three things:

- How much you borrow.
- How long you keep the money before paying it back.
- The interest rate, which is a percent.

For example: How much will you pay in interest on $2,000 for 3 years at 11.5%?

1. Change the percent to a decimal. 11.5% =.115
2. Multiply the **principal** by the **rate**. $2000 x .115 = $230 for 1 year
3. Multiply the interest for 1 year by 3 years. $230 x 3 = $690
4. The interest is $690 for three years at 11.5% interest.

The amount of money you borrow, or the amount you deposit is called **principal**. It is a dollar amount. The **interest rate** is a percent of the principal. It is based on a period of one year.

More interest problems:

1. How much will you pay in interest on a $5000 loan for 5 years at 5.5%? How much will you pay overall?
   a. Change the percent to a decimal. 5.5% = 0.055
   b. Multiply the **principal** by the **rate**. $5000 x 0.055 = $275
   c. Multiply the interest for 1 year by 5 years. $275 x 5 = $1375
   d. The interest is $1375 for five years at 5.5% interest.
   e. The total cost of the loan is $6375.

2. You would like to buy a new laptop. The cost of the laptop is $1500 with all the program software installed. You only have $500. The store offers you a loan for one year for the remainder of the money at a 8.9% interest rate. How much will you pay in interest? How much will you pay overall?

   a. How much money will the loan be for? $1500 - $500 = $1000

   b. Change the percent to a decimal. 8.9% = 0.089

   c. Multiply the principal by the rate. $1000 x 0.089 = $89

   d. The interest is $89 for one year at a 8.9% interest rate.

   e. The total cost of the laptop is $1589.00.

Other Important Math Concepts

Ratio

A **ratio** is a **comparison** of one number with another. It is used to show the relationship between something and something else.

The order of the numbers in a ratio is important. When you write a ratio, you must keep in mind which number belongs to which thing. A ratio can be written three ways:

1. With the word "to" 3 to 5
2. With a colon 3:5
3. As a fraction 3/5

Remember: When ratios are written as fractions, they are usually reduced to their lowest terms, even improper fractions, but you should not change an improper fraction to a whole number or a mixed number.

For example: Suppose you come into a room and see that there are 15 people but only 10 chairs.

The ratio of people to chairs is:

1. 15 to 10 ÷ 5/5 = 3 to 2
2. 15 : 10 ÷ 5/5= 3 : 2
3. $\frac{15}{10} \div \frac{5}{5} = \frac{3}{2}$

The ratio of chairs to people is 10 to 15 or 2 to 3.

Proportions

A **proportion** is a statement that **two ratios are equal.** This is how it is shown:

$$\frac{15}{10} = \frac{3}{2} \quad \text{or} \quad 15{:}10 = 3{:}2$$

It is read: " fifteen is to ten as three is to two". If proportions are equal, cross-multiply and the answer should be the same. 15 x 2 = 10 x 3

Cross-multiplication can be used to find an unknown number in a proportion.

For example: 5 is to 10 as 20 is to an unknown number.

1. Write the proportion, letting "*a*" stand for the unknown number.

$$\frac{5}{10} = \frac{20}{a}$$

2. Cross-multiply.

$5 \text{ X } a = 10 \times 20$
$5a = 200$
$a = 200 \div 5$
$a = 40$

Basic Exponents

Exponents are shorthand for repeated multiplication of the same thing by itself. For example: (5)(5)(5) = 5^3. The "exponent", being 3 in this example, stands for however many times the value is being multiplied. The thing that's being multiplied, being 5 in this example, is called the "base".

This process of using exponents is called "raising to a power", where the exponent is the "power". The expression "5^3" is pronounced as "five, raised to the third power" or "five to the third". There are two specially-named powers: "to the second power" is generally pronounced as "squared", and "to the third power" is generally pronounced as "cubed". So "5^3" is commonly pronounced as "five cubed".

Exponents have a few rules that we can use for simplifying expressions.

1. **Simply 10^4 = 10 x 10 x 10 x 10**

2. **Multiply $(x^3)(x^4) = x^7$**

   Whenever you multiply two terms with the same base, you can add the exponents.

3. **Divide $(x^4) \div (x^3) = x^1$**

   The rule for dividing exponents is to subtract the exponents (4-3 = 1) if you have the same base.

Metric Units and Measurement[1]

Length

The standard unit of length in the metric system is the metre. Other units of length and their equivalents in metres are as follows:

1 millimetre = 0.001 metre	We symbolize these lengths as follows:
1 centimetre= 0.01 metre	1 millimetre = 1 *mm*
1 decimetre = 0.1 metre	1 centimetre = 1 *cm*
1 kilometre = 1000 metres	1 metre = 1 *m*
100 centimetres = 1 metre	1 decimetre = 1 *dm*
1000 millimetres = 1 metre	1 kilometre = 1 *km*

For reference, 1 metre is a little longer than 1 yard or 3 feet. It is about half the height of a very tall adult. A centimetre is nearly the diameter of a dime, a little less than half an inch. A millimetre is about the thickness of a dime.

Volume

The standard unit of volume in the metric system is the litre. One litre is equal to 1000 cubic centimetres in volume. Other units of volume and their equivalents in litres are as follows:

1 millilitre = 0.001 litre	We symbolize these volumes as follows:
1 centilitre = 0.01 litre	1 millilitre = 1 *ml*
1 decilitre = 0.1 litre	1 centilitre = 1 *cl*
1 kilolitre = 1000 litres	

[1] From: http://www.mathleague.com/help/metric/metric.htm

1000 mililitres = 1 litre 1,000 litres = 1 cubic metre (m^3)	1 decilitre = 1 *dl* 1 litre = 1 *l* 1 kilolitre = 1 *kl*

For reference, 1 litre is a little more than 1 quart. One teaspoon equals about 5 millilitres.

Mass

The standard unit of mass in the metric system is the gram. Other units of mass and their equivalents in grams are as follows:

1 milligram = 0.001 gram 1 centigram = 0.01 gram 1 decigram = 0.1 gram 1 kilogram = 1000 grams	We symbolize these masses as follows: 1 milligram = 1 *mg* 1 centigram = 1 *cg* 1 decigram = 1 *dg* 1 gram = 1 *g* 1 kilogram = 1 *kg*

For reference, 1 gram is about the mass of a paper clip. One kilogram is exactly the mass of a litre of water.

Time

The following conversions are useful when working with time:

- 1 minute = 60 seconds
- 1 hour = 60 minutes = 3600 seconds
- 1 day = 24 hours

- 1 week = 7 days
- 1 year = 365 1/4 days (for the Earth to travel once around the sun)

In practice, every three calendar years will have 365 days, and every fourth year is a "leap year", which has 366 days, to make up for the extra quarter day over four years.

The year is divided into 12 months, each of which has 30 or 31 days, except for February, which has 28 days (or 29 days in a leap year).

Temperature

The USA and Imperial systems measure temperature using the Fahrenheit system. The Metric (SI) system originally used the Celsius temperature system, but now officially uses the Kelvin temperature system. However, few people aside from scientists have switched to the Kelvin system and the Celsius system is almost always used by most people for ordinary (non-scientific) purposes.

- The freezing point of water in Fahrenheit is 32 degrees, in Celsius it is 0 degrees.
- The boiling point of water in Fahrenheit is 212 degrees, in Celsius it is 100 degrees.
- Consequently the difference between freezing and boiling is 180 degrees Fahrenheit (212-32) or 100 degrees Celsius (100-0). This means that 180 degrees change in Fahrenheit is equal to 100 degree change in Celsius, or more simply 1.8 degrees Fahrenheit equals 1.0 degrees Celsius.

This gives rise to the following equations to convert between Celsius and Fahrenheit:

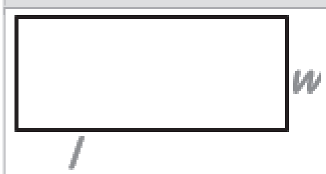
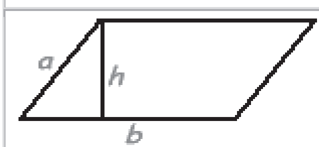
- **°C = (°F - 32) ÷ 1.8** For example: (68°F-32) ÷ 1.8 = (36) ÷ 1.8 = 20°C
- **°F = (°C x 1.8) + 32** For example: (20°C x 1.8) + 32 = (36) + 32 = 68°F

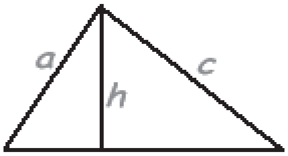
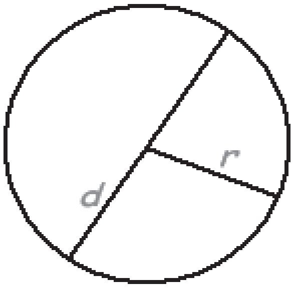
Some common examples are:

- Freezing = **0 °C , 32°F**
- Room temperature = **20 °C , 68°F**
- Normal body temperature = **37 °C , 98.6°F**
- A very hot day = **40 °C , 104°F**
- Boiling point of water = **100 °C , 212°F**

Measurement

Area is the amount of two-dimensional space. Area is also used to measure the outermost surface of an object. Perimeter is the; total length of the outer boundary.

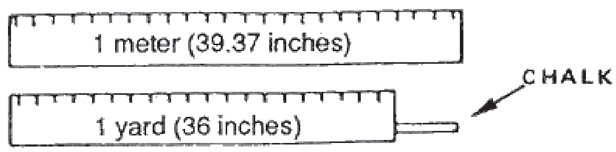
Shapes	Formula
	**Rectangle:** Area = Length X Width A = lw Perimeter = 2 X Lengths + 2 X Widths P = 2l + 2w
	**Parallelogram** Area = Base X Height a = bh Perimeter = 2 X Lengths + 2 X Widths P = 2l + 2w

	**Triangle** Area = 1/2 of the base X the height a = 1/2 bh Perimeter = a + b + c (add the length of the three sides)
	**Circle** The distance around the circle is a circumference. The distance across the circle is the diameter (d). The radius (r) is the distance from the center to a point on the circle. (π = 3.14) diameter = 2r circumference= πd = 2 πr Area = πr^2 (π =3.14)

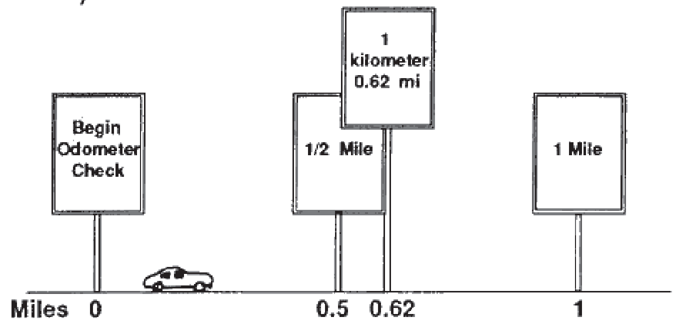
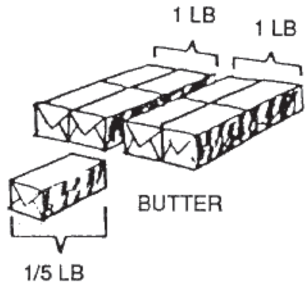
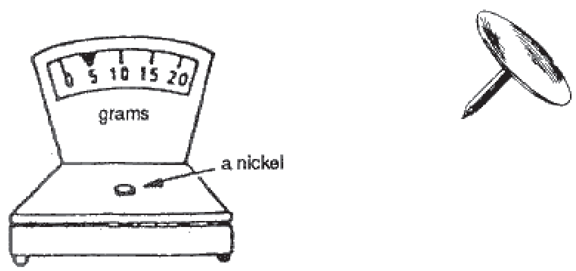
More on Length, Mass, Volume and Temperature

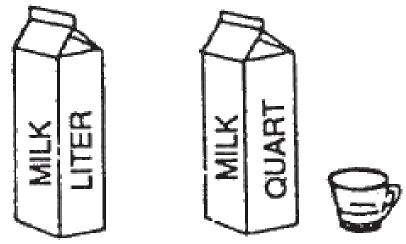
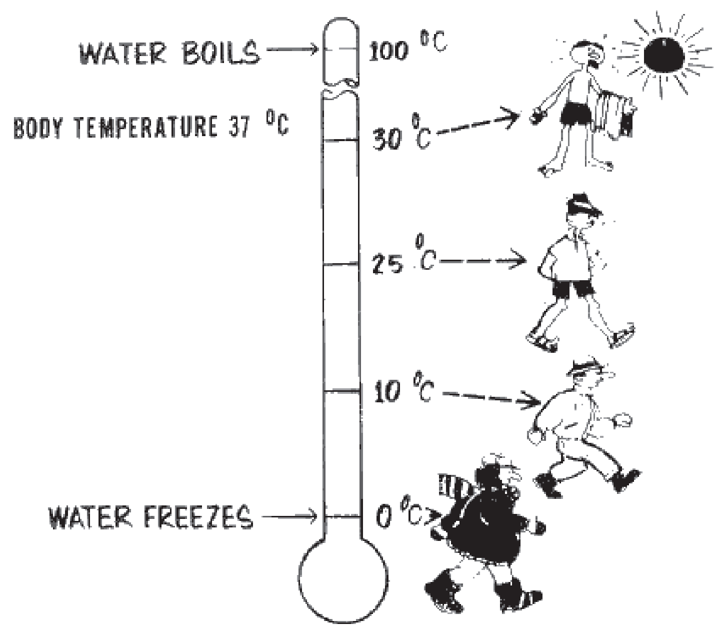
Look at the chart below to help you get a better understanding of the units, metre, centimetre, kilometre, kilogram, gram, litre, mililitre and Celsius.[2]

You might notice that the spelling for metre, centimetre, kilometre, litre and mililitre are different in the below chart. The American spelling for metre is meter and so forth.

Unit	Equivalence
Length: 1 metre (1 m)	**1 meter** **(or 1 m) = about a yardstick plus the length of a piece of chalk** 
1 centimetre (1 cm)	**1 centimeter (or 1 cm) = the width of some part of your smallest finger or fingernail** 

[2] From: http://lamar.colostate.edu/~hillger/frame.htm

1 kilometre (1 km)	**1 kilometer** (or 1 km) = a little more than **half a mile** (pronounced KILL-oh-meet-ur not kill-AHM-it-ur) 
Mass: 1 kilogram (1 kg)	**1 kilogram (or 1 kg) = about the mass of 2.2 pounds of butter** 
1 gram (1 g)	**1 gram (or 1 g) = about the mass of a large thumbtack**  **a nickel = about 5 grams (or 5 g)**

Volume: 1 liter (1 L or 1 l)	**1 liter (or 1 L or 1 l)** EQUALS **1 quart plus 1/4 cup = 1 liter** 
1 millilitre (1 mL or 1 ml)	**1 milliliter (or 1 mL or 1 ml) = 1/5 tsp**  **1 tsp = 5 milliliters (mL)** 
Temperature: degree Celsius (°C)	**Metric Temperature (degree Celsius)** 

Metric Vs. Imperial: Conversion Charts and Information

Fact: The United States is the only industrialized country in the world that doesn't use the metric system as its predominant system of weights and measures. Today only the USA, Liberia and Myanmar still use the old English Imperial system. The rest of the world is metric.

Given that Canada is a neighbor of the USA we still use quite a bit of the imperial system. For example do you weigh yourself in kilograms or pounds? Do you measure land in acres or hectres? Do you measure yourself in feet and inches or centimetres?

Imperial Unit	Metric (SI) Unit
Inch	2.54 centimetres
Foot	30.48 centimetres
Yard	0.91 metres
Mile	1.61 kilometres

Metric (SI) unit	Imperial Unit
Centimetre	0.39 inches
Metre	3.28 feet
Metre	1.09 yards
Kilometre	0.62 miles

Imperial/USA unit	Metric (SI) unit
Ounce (weight)	28.35 grams
Pound	0.45 kilograms
UK ton (2240 pounds)	1.02 metric tons
US ton (2000 pounds)	0.91 metric tons

Metric (SI) unit	Imperial/USA unit
Gram	0.035 ounces
Kilogram	2.21 pounds
Metric ton (1000 kg.)	0.98 UK tons
Metric ton (1000 kg.)	1.10 US tons

Imperial/USA unit	Metric (SI) unit
Acre	0.40 hectare
Square inch	6.45 square centimetres
Square foot	0.09 square metres
Square yard	0.84 square metres
Square mile	2.60 square kilometres
Cubic foot	0.028 cubic metres
Cubic yard	0.76 cubic metres

Metric (SI) unit	Imperial/USA unit
Hectare	2.47 acres
Square centimetre	0.16 square inches
Square metre	1.2 square yards
Square metre	1.20 square yards
Square kilometre	0.39 square miles
Cubic metre	35.23 cubic feet
Cubic metre	1.35 cubic yards

Imperial/USA unit	Metric (SI) unit
Teaspoon (UK)	5.92 millilitres
Teaspoon (US)	4.93 millilitres
Tablespoon (UK)	17.76 millilitres
Tablespoon (US)	14.79 millilitres
Fluid ounce (UK)	28.41 millilitres
Fluid ounce (US)	29.57 millilitres
Pint (UK)	0.57 litres
Pint (US)	0.47 litres
Quart (UK)	1.14 litres
Quart (US)	0.95 litres
Gallon (UK)	4.55 litres
Gallon (US)	3.79 litres

Metric (SI) unit	Imperial/USA unit
Millilitre	0.17 teaspoons (UK)
	0.20 teaspoons (US)
10 Millilitre	0.56 tablespoons (UK)
	0.68 tablespoons (US)
100 millilitre	3.52 fluid ounces (UK)
	3.38 fluid ounces (US)
Litre	1.76 pints (UK)
	2.11 pints (US)
	0.88 quarts (UK)
	1.06 quarts (US)
	0.22 gallon (UK)
	0.26 gallons (US